Lattice benchmark observables:
- axial charge of the nucleon $g_A$
- quark average momentum in nucleon $\langle x \rangle_{u-d}$
- form factors $G_E, G_M$

scalar quark content of the nucleon

Going further
- Gluon moment
- nucleon spin

Summary and outlook
Quark parton distribution functions

- $e^{-}$: momentum $p$
- $p$: momentum $q$
- $\gamma$: momentum transfer $Q^2$
- Bjorken variable: $x = \frac{Q^2}{2pq}$

→ inaccessible to direct lattice calculations (see, however, recent attempts, Ji)

but, can compute moments of quark parton distribution functions $q(x, \mu^2)$

$$\langle x^n \rangle_{q, \mu^2} = \int dx x^n q(x, \mu^2)$$

→ moments $\langle x^n \rangle_{q, \mu^2}$ can be related to local operators
Examples of local operators

- **vector operator**

\[
\mathcal{O}^{\mu_1...\mu_n}_{V a} = \bar{\psi} \gamma^{\{\mu_1} D^{\mu_2} \ldots D^{\mu_n\}} \tau^a \psi
\]

- **axial vector operator**

\[
\mathcal{O}^{\mu_1...\mu_n}_{A a} = \bar{\psi} \gamma^{\{\mu_1} D^{\mu_2} \ldots D^{\mu_n\}} \gamma^5 \tau^a \psi
\]

- **axial charge of the nucleon**, i.e. \( n = 1 \) for axial operator

\[
\mathcal{O}_{g A} = \bar{\psi} \gamma_\mu \gamma^5 \tau^3 \psi
\]

- **average quark momentum in nucleon** \( n = 2 \) for vector operator

\[
\mathcal{O}_{<x>} = \bar{\psi} \gamma_\mu \gamma_\nu \tau^3 \psi
\]
Form factor decomposition

vector case

\[ \langle N(p', s') | O_{V3}^\mu | N(p, s) \rangle = \bar{u}_N(p', s') \left[ A_{20}(q^2) \gamma^{\mu} P^\nu + B_{20}(q^2) \frac{i\sigma^{\mu\alpha} q_{\alpha} P^\nu}{2m} + C_{20}(q^2) \frac{1}{m} q^{\mu} q^{\nu} \right] \frac{1}{2} u_N(p, s) \]

axial case

\[ \langle N(p', s') | O_A^\mu | N(p, s) \rangle = \bar{u}_N(p', s') \left[ G_A(q^2) \gamma^{\mu} \gamma_5 + \frac{q^{\mu} \gamma_5}{2m_N} G_p(q^2) \right] \frac{1}{2} u_N(p, s) \]

- axial charge \( g_A = G_A(q^2 = 0) \)
- average momentum \( \langle x \rangle_{u-d} = A_{20}(q^2 = 0) \)
- quark contribution to nucleon spin \( J^q = \frac{1}{2} [A_{20}(0) + B_{20}(0)] \)
Evaluation of nucleon 2-point function

→ see also introductory lecture by Giannis Koutsou

nucleon interpolating field:

\[ J(x) = \epsilon^{abc} [u^a(x) C \gamma_5 d^b(x)] u^c(x) \]

nucleon 2-point function (neglecting boundary conditions)

\[ G_2(t_f - t_i) = \sum_{\bar{x}_i, \bar{x}_f} \Gamma^0_{\beta \alpha} \langle J_\alpha(t_f, \bar{x}_f) \bar{J}_\beta(t_i, \bar{x}_i) \rangle = \sum_n e^{-m_n(t_f - t_i)} \]

\[
\lim_{(t_f-t_i) \to \infty} G_2(t_f - t_i) \to J^{(0)}_N \bar{J}^{(0)}_N e^{-m_N(t_f - t_i)}
\]

\[ E_{\text{eff}} = -\frac{1}{t_f - t_i} \log(G_2(t_f - t_i)) \]
The lattice QCD benchmark calculation: the spectrum

spectrum for $N_f = 2 + 1$ and $2 + 1 + 1$ flavours

first spectrum calculation BMW repeated by other collaborations

- spectrum also for $N_f = 2 + 1 + 1$ flavours ETMC (Cyprus & DESY Zeuthen)
- only two input parameters
- roper: still unresolved
Charmed baryons
(C. Alexandrou, V. Drach, K. Hadiyiannakou, C. Kallidonis, K.J.)

- we can go further (← only two additional inputs)
Even isospin and electromagnetic mass splitting

(BMW collaboration)

baryon spectrum with mass splitting

- nucleon: isospin and electromagnetic effects with opposite signs
- nevertheless physical splitting reproduced
Evaluation of 3-point function

\[ q = p' - p \]

\[ G_{\mu_1 \ldots \mu_n}(t_f, t_i, t) = \sum \bar{x}, \bar{x}_i, \bar{x}_f \Gamma_{\beta \alpha}^{\nu} \langle J_{\alpha}(t_f, \bar{x}_f) \mathcal{O}^{\mu_1 \ldots \mu_n}(t, \bar{x}) \bar{J}_{\beta}(t_i, \bar{x}_i) \rangle \]

\[ \lim_{(t_f - t_i), t \to \infty} G_3(t_f, t_i, t) \to J_N^{(0)} \bar{J}_N^{(0)} e^{-m_N(t_f - t_i)} \langle 0 | \hat{O} | 0 \rangle \]

in infinite time limit

\[ \langle 0 | \mathcal{O} | 0 \rangle = \lim_{(t_f - t_i), t \to \infty} \frac{G_3(t_f, t_i, t)}{G_2(t_f - t_i)} \]
Example: iso-vector and scalar axial charge $g_A$

- look for a plateau value
- probe different insertion times
Evaluation of 3-point function, dis-connected part

- have developed technology to compute dis-connected diagrams

example dis-connected contribution to isoscalar axial charge $g_A$

- can identify a signal
- find only small contribution $O(10\%)$ of connected
Non-perturbative renormalization

- RI-MOM scheme
- High precision through momentum sources
- Perturbative $O(a^2)$ subtraction

\[ Z_\Gamma G(p^2)|_{p^2=\mu^2} = G^{\text{tree}}(p^2)|_{p^2=\mu^2} \]

- Blue points: original data
- Black points: perturbative $O(a^2)$ subtraction
- Perturbative conversion to $\overline{\text{MS}}$ scheme
Players in the games

**ETMC, Cyprus-DESY**
C. Alexandrou, M. Constantinou, S. Dinter, V. Drach, K. Hatziyiannakou, K. Jansen, C. Kallidonis, G. Koutsou, T. Leontiou, A. Vaquero C. Wiese

**Mainz/CLS**

**PNDME**
T. Bhattacharya, S.D. Cohen, R. Gupta, A. Joseph, H.-W. Lin

**LHPC**

**RBC-UKQCD**

**Regensburg, JSC**

**QCDSF**
R. Horsley, Y. Nakamura, A. Nobile, P.E.L. Rakow, G. Schierholz, J.M. Zanotti

**MIT group**
J. R. Green, M. Engelhardt, S. Krieg, J. W. Negele, A. V. Pochinsky, S. N. Syritsyn
benchmark observables: $g_A$

(D. Renner, Lattice’09)

axial charge

- about $O(-10\%)$ deviation

(C. Alexandrou, M. Constantinou, S. Dinter, V. Drach, D. Renner, K.J.)

relative deviation
benchmark observables $\langle x \rangle_{u-d}$

(D. Renner, Lattice'09)

average quark momentum

- about $O(20-30\%)$ deviation

(C. Alexandrou, M. Constantinou, S. Dinter, V. Drach, D. Renner, K.J.)

relative deciation
Dirac radius

QCDSF
(“Pohl”: myonic hydrogen measurement)

- discrepancy at the O(50%) level
Systematic uncertainties

⋆ ⋆ ⋆ results of world-wide lattice collaborations compatible

- lattice spacing artefacts
- finite volume effects
- excited state contributions
- extrapolations
  - chiral extrapolation to physical pion mass
  - $Q^2 \to 0$ extrapolation in form factors
Excited state effects

reminder: nucleon 2-point function (use $\Delta t = t_f - t_i$)

$$G_2(\Delta t) = \sum_{\vec{x}_i, \vec{x}_f} \Gamma_{\beta\alpha}^0 \langle J_\alpha(t_f, \vec{x}_f) J_\beta(t_i, \vec{x}_i) \rangle = \sum_n e^{-m_n(\Delta t)}$$

for 3-point function:

$$\frac{C_3(\Delta t, t)}{C_2(\Delta t)} = \langle 0 | \mathcal{O} | 0 \rangle$$

$$+ \langle 0 | \mathcal{O} | 1 \rangle \frac{\bar{J}_N^{(1)}}{J_N^{(0)}} \exp (-\Delta M t)$$

$$+ \langle 1 | \mathcal{O} | 0 \rangle \frac{J_N^{(1)}}{J_N^{(0)}} \exp [-\Delta M (\Delta t - t)]$$

$$+ \mathcal{O} [\exp (-\Delta M \Delta t)]$$

• $\Delta M$ difference between nucleon mass and next excited state
dedicated effort: $g_A$, $\langle x \rangle_{u-d}$

(ETMC, PNDME)

7500 measurements
(standard: $\approx 500$ measurements)

no excited state effects $g_A$

23000 measurements

excited states $\approx O(10\%)$ of deviation
(remember: deviation larger than 20% )

result from ETMC, consistent with PNDME
Costs of dynamical fermions simulations, the “Berlin Wall”

see panel discussion in Lattice2001, Berlin, 2001

formula \( C \propto \left( \frac{m_\pi}{m_\rho} \right)^{-z_\pi} (L)^{z_L} (a)^{-z_a} \)

\[ z_\pi = 6, \quad z_L = 5, \quad z_a = 7 \]

“both a 10^8 increase in computing power AND spectacular algorithmic advances before a useful interaction with experiments starts taking place.”
(Wilson, 1989)

⇒ need of Exaflops Computers

physical contact to \( \chiPT (?) \)
A generic improvement for Wilson type fermions

New variant of HMC algorithm (Urbach, Shindler, Wenger, K.J.)
(see also SAP (Lüscher) and RHMC (Clark and Kennedy) algorithms)

- even/odd preconditioning
- (twisted) mass-shift (Hasenbusch trick)
- multiple time steps

![Graph showing T-ops per year vs. m_Ps / m_v for different lattice sizes and inverse lattice spacings.](image)

1000 configurations with L=2fm
[Ukawa (2001)]

---

2001

2006
Simulation landscape
(thanks to G. Herdoiza)

<table>
<thead>
<tr>
<th>Collaboration</th>
<th>$N_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLS</td>
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</tr>
<tr>
<td>ETMC</td>
<td>2</td>
</tr>
<tr>
<td>QCDSF</td>
<td>2</td>
</tr>
<tr>
<td>BGR</td>
<td>2</td>
</tr>
<tr>
<td>JLQCD</td>
<td>2</td>
</tr>
<tr>
<td>TWQCD(plaq)</td>
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</tr>
<tr>
<td>TWQCD(Iwa)</td>
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</tr>
<tr>
<td>BMW(HEX)</td>
<td>$2 + 1$</td>
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<tr>
<td>BMW(stout)</td>
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<td>MILC</td>
<td>$2 + 1$</td>
</tr>
<tr>
<td>MILC</td>
<td>$2 + 1 + 1$</td>
</tr>
<tr>
<td>ETMC</td>
<td>$2 + 1 + 1$</td>
</tr>
</tbody>
</table>

note: all used actions $O(a^2)$-continuum limit scaling
An example for a physical point simulation: baryon mass spectrum from ETMC

- lattice spacing: $a \approx 0.095\text{fm}$
- lattice volume: $48^3 \cdot 96$
- pion mass: $m_\pi \approx 130\text{GeV}$
The nucleon axial charge at the physical point

- eliminate chiral extrapolation effect
- physical point results
  → conflict between collaborations

- finite volume effects?
- discretization effects?
- excited state effects?
- need higher statistics
Average quark momentum at the physical point

- eliminate chiral extrapolation effect
- trend towards physical result

- finite volume effects?
- discretization effects?
- excited state effects?
- need more statistics
Excited states at the physical point?
(M. Engelhardt, J. R. Green, S. Krieg, J. W. Negele, A. V. Pochinsky, S. N. Syritsyn)

excited state effects

excited states go towards physical result

excited states go away from physical result
The epoche of physical point simulations

- eliminate systematic (severe) uncertainty of chiral extrapolation
- have to attack remaining systematic errors
  - finite volume effects
  - discretization effects
  - excited state contributions
  - non-perturbative renormalization

⇒ need of even more advanced and demanding simulations
More observables queueing: nucleon form factor

- Overall agreement with experiment
The nucleon electric form factor

\[ G_E(Q^2) = F_1(Q^2) + F_2(Q^2) \]

- difficult to match experimental electric form factor data
  (very preliminary: difficulty seems to remain at physical point)
The strange quark content of the nucleon  
(C. Alexandrou, M. Constantinou, V. Drach, S. Dinter, R. Frezzotti,  
K. Hadjiyiannakou, G. Herdoiza, G. Koutsou, G. Rossi, A. Vaquero, K.J.)

- neutralino in supersymmetric models candidate for dark matter

- interaction with nucleon most strongly through the strange quark content  
  via the Higgs boson exchange diagram

\[
\sigma_{SI} \propto \left( \sum_q f_{Tq} \right)^2 ; q = u, d, s, c
\]

\[
f_{Tq} = \frac{\langle N | m_q \bar{q} q | N \rangle}{m_N}
\]

⇒ cross section proportional to quark content; independent from quark mass
The problem

Spin independent cross section strongly depend on pion-nucleon sigma term $\sigma_{\pi N}$

Varying $48\text{MeV} < \sigma_{\pi N} < 80\text{MeV}$

$\Rightarrow$ cross section changes by an order of magnitude

Estimates for $\sigma_{\pi N}$:

$\sigma_{\pi N} = 45 \pm 8 \text{ MeV}$ (GLS) (J. Gasser, H. Leutwyler, M. Sainio)

$\sigma_{\pi N} = 64 \pm 7 \text{ MeV}$ (GWU) (M. Pavan, I. Strakovsky, R. Workman, R. Arndt)

$\sigma_{\pi N} = 59 \pm 7 \text{ MeV}$ (AMO) (J. Alarcon, J. Martin Camalich, J. Oller)
The $y_N$ parameter

$\sigma_{\pi N}$ connected to $y_N$ parameter

$$y_N \equiv \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u+\bar{d}d|N\rangle}$$

$\rightarrow y_N$ measures the strange quark content of nucleon

relation: $y_N = 1 - \sigma_0/\sigma_{\pi N}$

$$\sigma_0 = m_q\langle N|\bar{u}u + \bar{d}d - 2\bar{s}s|N\rangle, \quad m_q = (m_u + m_d)/2$$

using $\sigma_0 = 36 \pm 7$ MeV (B. Borasoy, U.-G. Meissner)

$$y_N^{\text{GLS}} = 0.20(21), \quad y_N^{\text{GWU}} = 0.44(13), \quad y_N^{\text{AMO}} = 0.39(14)$$

- values rather large and affected by large errors
- want: a first principle, non-perturbative computation
Constraints on $\sigma$-terms: a twisted mass (Cyprus-DESY) analysis

- special noise reduction techniques for dis-connected graphs
- twisted mass: avoid mixing in renormalization
- obtain value: $y_N = 0.135(22)(33)(22)(9)$
  errors: statistical, chiral extrapolation, excited states, discretization

- constraining strange $\sigma$-term: $\sigma_s = y_N \frac{1}{2} m_s \sigma_{\pi N} \lesssim 200 \text{MeV}$
- direct calculation: went to 150 000 measurements
Comparing sigma terms

**strange \( \sigma \)-term**

- overall agreement of smallish value
- start to attack systematic errors

**\( y_N \) parameter**

---

Phenomenological determinations

GLS
GWU
AMO

JLQCD – \( N_f = 2 + 1 \)

Young et al – Indirect method – LO SU(3) – \( N_f = 2 + 1 \)
BMW – Indirect method – \( N_f = 2 + 1 \)
Horsley et al. – Indirect method – \( N_f = 2 + 1 \)

Semke et al. – Indirect method – LO SU(3) – \( N_f = 2 + 1 \)
Shanahan et al. – Indirect method – LO SU(3) – \( N_f = 2 + 1 \)

Ren et al. – Indirect method – LO SU(3) – \( N_f = 2 + 1 \)
Junnarkar et al. – Indirect method – \( N_f = 2 + 1 \)
JLQCD – \( N_f = 2 + 1 \)
Engelhardt – \( N_f = 2 + 1 \)
Freeman et al. – \( N_f = 2 + 1 \)
ETM [this work] – Direct method – \( N_f = 2 + 1 + 1 \)

Quantum Chromodynamics

QCDSF – Direct method – \( N_f = 2 \)

QCDSF/UKQCD – Indirect method – \( N_f = 2 + 1 \)
BMW – Indirect method – \( N_f = 2 + 1 \)

ETM [this work] – Direct method – \( N_f = 2 + 1 + 1 \)
Gluon moment
( C. Wiese, C. Alexandrou, M. Constantinou, V. Drach, K. Hadjiyiannakou, B. Kostrzewa, H. Panagopoulos, K.J. )

• gluon operator:
  compute matrix elements with a ratio of three-point and two-point function:

\[
\frac{\langle H(p,t)\mathcal{O}(\tau)H(p,0) \rangle}{\langle H(p,t)H(p,0) \rangle} \quad \text{for} \quad 0 \ll \tau \ll t \quad = \quad \langle \mathcal{O} \rangle H(p)H(p) , \quad \mathcal{O}_{\mu\nu} = -\text{tr}_c G_{\mu\rho} G_{\nu\rho}
\]

• smear the gauge fields (Meyer, Negele)

\[ \text{plateau for } t_s = 12 \]

• see a signal \( \rightarrow \) promising
A comprehensive study of techniques for evaluating dis-connected diagrams

<table>
<thead>
<tr>
<th>Method</th>
<th>Abs. Error</th>
<th>Cost</th>
<th>Cost $\times$ Error$^2$</th>
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<tbody>
<tr>
<td>$\sigma_{\pi N}$</td>
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<td></td>
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<tr>
<td>One-end trick</td>
<td>0.0043 GeV</td>
<td>2234</td>
<td>0.032</td>
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<tr>
<td>One-end trick + TSM</td>
<td>0.0038 GeV</td>
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<td>$\sigma_{s}$</td>
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<td>One-end trick</td>
<td>0.0051 GeV</td>
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<td>0.019</td>
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<tr>
<td>One-end trick + TSM</td>
<td>0.0049 GeV</td>
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<tr>
<td>Time-dilution + HPE</td>
<td>0.0080 GeV</td>
<td>750</td>
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<tr>
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<td>$\sigma_{c}$</td>
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<td>$g_{A}$</td>
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<tr>
<td>$g_{A}^{s}$</td>
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<td>0.076</td>
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<td>Time-dilution</td>
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<td>Time-dilution + HPE</td>
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<td>One-end trick + TSM</td>
<td>0.0215</td>
<td>692</td>
<td>0.32</td>
</tr>
</tbody>
</table>

- No overall best method
- Test of each observable individually
The question of the spin

- Results for nucleon spin for larger than physical pion masses
- Difficult to extrapolate → take result at lowest pion mass
# A summary of the nucleon spin

<table>
<thead>
<tr>
<th></th>
<th>(m_\pi = 213) MeV</th>
<th>experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>(J^{u-d})</td>
<td>0.217(32)</td>
<td></td>
</tr>
<tr>
<td>(J^{u+d})</td>
<td>0.211(30)</td>
<td></td>
</tr>
<tr>
<td>(J^u)</td>
<td>0.214(27)</td>
<td></td>
</tr>
<tr>
<td>(J^d)</td>
<td>-0.003(17)</td>
<td></td>
</tr>
<tr>
<td>(\Delta \Sigma^{u-d}/2)</td>
<td>0.582(31)</td>
<td>0.634(2)</td>
</tr>
<tr>
<td>(\Delta \Sigma^{u+d}/2)</td>
<td>0.303(26)</td>
<td>0.208(9)</td>
</tr>
<tr>
<td>(\Delta \Sigma^u/2)</td>
<td>0.443(24)</td>
<td>0.421(6)</td>
</tr>
<tr>
<td>(\Delta \Sigma^d/2)</td>
<td>-0.140(16)</td>
<td>-0.214(6)</td>
</tr>
<tr>
<td>(L^{u-d})</td>
<td>-0.365(45)</td>
<td></td>
</tr>
<tr>
<td>(L^{u+d})</td>
<td>-0.092(41)</td>
<td></td>
</tr>
<tr>
<td>(L^u)</td>
<td>-0.229(30)</td>
<td></td>
</tr>
<tr>
<td>(L^d)</td>
<td>0.137(30)</td>
<td></td>
</tr>
</tbody>
</table>

- only result at smallest pion mass
- only one lattice spacing and volume
A hope for the future

<table>
<thead>
<tr>
<th>Collaboration</th>
<th>$m_{\text{phys}}$</th>
<th>$a \to 0$</th>
<th>$L \to \infty$</th>
<th>$e^{-\Delta E t} \ll 1$</th>
<th>non-pert. renorm.</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>★ ★ ★</td>
<td>★ ★</td>
<td>★</td>
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<td>★ ★ ★</td>
<td>★ ★ ★</td>
<td>★</td>
</tr>
</tbody>
</table>

- want 3 green stars everywhere
Summary and outlook

• present situation for $m_\pi \lesssim 210$ MeV
  
  – striking agreement between different lattice groups
  – nevertheless tension/discrepancies with experiment/phenomenology
  – world looks really different at unphysical pion masses

• entering a new epoch of lattice calculations:
  *computations at physical pion mass have started*
  
  – very promising, but too early to conclude
  – study of other systematic effects essential
  – situation not yet clarified ($g_A$, $\langle x \rangle_{u-d}$, electric form factor)

• more exciting prospects
  
  – have developed efficient methods for dis-connected diagrams
  – all mode averaging seems to work very successfully
  \[ \rightarrow \] prospect of substantially increased accuracy