Πανεπιστήμιο Κύπρου
University of Cyprus

Book of Abstracts

HOFEIM 2023

9th International Conference on High Order Finite Element and Isogeometric Methods

29 May – June 1

Larnaca, Cyprus

HOFEIM 2023

The ***9th International Workshop on High-Order Finite Element and Isogeometric Methods*** (HOFEIM 2023) will be held in Larnaca, Cyprus from May 29 to June 1, 2023.

In the last decades p-, hp-FEMs and Isogeometric Analysis (based on splines, NURBS, and extensions) witnessed a rapid growth in their use for numerical simulation in many relevant areas, such as computational mechanics, fluid dynamics, electromagnetism and waves propagation. HOFEIM 2023 will be dedicated to the recent developments of these methods, bringing together researchers with interests in their mathematical foundations as well as in their applicability to engineering practice. The aim of the workshop is twofold: to update experienced researchers in the area, as well as to form young researchers interested in using these techniques in their research activities.

Scientific Committee

- Yuri Bazilevs, Brown University, USA
- Leszek Demkowicz, Univ. of Texas at Austin, USA
- Alexander Düster, Hamburg Univ. of Technology, Germany
- Tom Hughes, University of Texas at Austin, USA
- Stefan Kollmannsberger, Technische Universität München, Germany
- Markus Melenk, Technische Universität Wien, Austria
- Barna Szabó, Washington University, St. Louis, USA
- Ernst Rank, Technical University of Munich, Germany
- Alessandro Reali, University of Pavia, Italy
- Giancarlo Sangalli, University of Pavia, Italy
- Zohar Yosibash, Tel Aviv University, Israel

Local Organizing Committee

- Christos Xenophontos, University of Cyprus
- Georgios Georgiou, University of Cyprus

Message from the local organizers

It is with great pleasure that we welcome all the delegates and accompanying persons to the 9th International Conference on High Order Finite Element and Isogeometric Methods (HOFEIM 2023), Larnaca, Cyprus, May 29 – June 1, 2023.

HOFEIM 2023 is devoted to Professor Ivo Babuška (1926-2023) for his seminal contributions to the field. The program includes 40 regular talks and 4 poster presentations. With approximately 44 registered participants from fourteen countries (Austria, Cyprus, Finland, France, Germany, Israel, Italy, Netherlands, Poalnd, Saudi Arabia, Spain, Switzeralnd, UK, and USA) the conference is truly an international event.

We hope that *HOFEIM2023* will promote scientific exchange, collaboration and interactions between participants.

Christos Xenophontos and Georgios Georgiou

Programme

SUNDAY, MAY 28

Registration 18:30 – 19:00 outside "Ballroom"

Welcome Reception 19:00 – 20:30 at the pool area

MONDAY, MAY 29 – All presentations will take place in the room **Ballroom** of the hotel

08:15	–	**09:15**	**Registration**	
09:15	–	**09:30**	**Welcome and Opening Remarks**	
09:30	–	09:50	**Leszek Demkowicz**, Ivo Babuška tribute	**Chair** Zohar Yosibash
09:50	–	10:15	**Leszek Demkowicz**, Markus J. Melenk, Stefan Henneking, Jacob Badger, *Full envelope DPG approximation for electromagnetic waveguides. stability and convergence analysis*	
10:15	–	**10:40**	**Coffee Break**	
10:40	–	11:05	**Witold Cecot**, Marta Oleksy, Marek Klimczak. *Multiscale FEM and DPG methodology for upscaling in solid mechanics*	**Chair** Markus Melenk
11:05	–	11:30	**Jacob Badger**, Leszek Demkowicz, *Scalable hp-adaptive DPG multigrid solver with applications in high-frequency wave propagation*	
11:30	–	11:55	**Judit Muñoz-Matute**, Leszek Demkowicz, David Pardo, *The DPG method as a time-integration scheme for linear and non-linear transient PDEs*	
11:55	–	12:20	**Brendan Keith**, Thomas Surowiec, *The entropic finite element method*	
12:20	–	**14:20**	**Lunch**	
14:20	–	14:45	**Philipp Kopp**, Ernst Rank Victor Calo, Stefan Kollmannsberger, *Immersed space-time hp-finite elements for temperature evolution in laser powder bed fusion*	**Chair** Alexander Düster
14:45	–	15:20	**Marco Zank**, *Space-time continuous Galerkin methods for the wave equation*	
15:20	–	15:45	Paolo Bignardi, **Andrea Moiola**, *A space–time continuous and coercive formulation for the wave equation*	
15:45	–	**16:10**	**Coffee Break**	
16:10	–	16:35	**Massimo Carraturo**, *Modeling, calibration, and validation of powder bed fusion process simulations using the finite cell method*	**Chair** Giancarlo Sangalli
16:35	–	17:00	**Lisa Hug**, Stefan Kollmannsberger, Ernst Rank, *Adaptive phase-field simulations with the parallel finite cell method*	
17:00	–	17:25	**Paul Houston**, Matthew E. Hubbard, Thomas J. Radley, Oliver J. Sutton, Richard S.J. Widdowson, *hp-version polytopic discontinuous Galerkin methods for radiation transport Problems*	

17:25		17:50	**Théophile Chaumont-Frelet**, Axel Modave, *A hybridizable discontinuous Galerkin method with characteristic variables for high-frequency wave propagation problems*	

15:45	–	**17:50**	**Poster Presentations***
	–		**Balázs Tóth**, Alexander Düster, *Adaptive radial basis function finite difference scheme for linear elasticity problems* Christos Xenophontos, Sebastian Franz, **Irene Sykopetritou**, *Mixed hp finite element method for singularly perturbed fourth order boundary value problems with two small parameters* Hind Lamsikine, Otmane Souhar, **Georgios C. Georgiou**, *The singular function boundary integral method for solving three-dimensional Laplacian problems with conical vertex singularities* Christos Xenophontos, **Neofytos Neofytou**, *hp discontinuous Galerkin finite element methods for the approximation of singularly perturbed boundary value problems with two small parameters*

* Posters will be displayed in the afternoon of May 29 and remain throughout the conference.

TUESDAY, MAY 30 – All presentations will take place in the room **Ballroom**

09:00	–	09:25	**Alessandro Reali**, *Isogeometric analysis: advances and applications with a special focus on dynamic problems*	**Chair** Ernst Rank
09:25	–	09:50	**Monica Montardini**, Giancarlo Sangalli, Mattia Tani, *Low-rank solver for isogeometric analysis*	
09:50	–	10:15	**Mattia Tani**, Monica Montardini, *Fast Poisson solvers for isogeometric analysis*	
10:15	–	**10:40**	**Coffee Break**	
10:40	–	11:05	Gregor Gantner, **Martin Vohralík**, *Inexpensive polynomial-degree-robust equilibrated flux a posteriori estimates for isogeometric analysis*	**Chair** Stefan Kollmannsberger
11:05	–	11:30	**Andrea Bressan**, *Anisotropic refinement with LR-splines*	
11:30	–	11:55	**Matthias Möller**, *IgANets: Physics-informed machine learning embedded into isogeometric analysis*	
11:55	–	12:20	G. Loli, M. Montardini, **G. Sangalli**, M. Tani, *Space-time IGA*	

12:20	–	**14:20**	**Lunch**	
14:20	–	14:45	**Christoph Schwab**, Lehel Banjai, Markus Melenk, *Exponential convergence of hp FEM for spectral fractional diffusion in polygons*	**Chair** Christos Xenophontos
14:45	–	15:20	Markus Faustmann, **Carlo Marcati**, Jens Markus Melenk, Christoph Schwab, *Weighted analytic regularity for the integral fractional Laplacian in polygons*	
15:20	–	15:45	Markus Faustmann, Carlo Marcati, **Jens M. Melenk**, Christoph Schwab, *Exponential convergence of hp-FEM for the integral fractional Laplacian*	
15:45	–	**16:10**	**Coffee Break**	
16:10	–	16:35	**Andreas Schröder**, Paolo Di Stolfo, *hp-finite elements with higher differentiability on meshes with hanging nodes*	**Chair** Alessandro Reali
16:35	–	17:00	Cesare Bracco, **Carlotta Giannelli**, Mario Kapl, Rafael Vázquez, *High order hierarchical spline methods on multi-patch geometries*	
17:00	–	17:25	**Alexander Düster**, Wadhah Garhuom, *Improving the robustness of the finite cell method for nonlinear problems of solid mechanics*	
17:25	–	17:55	**Maciej Paszyński**, *Deep neural networks for smooth approximation of physics with higher order and continuity basis functions*	
20:30	–		**Conference Dinner** – Location: Elia Backyard	

WEDNESDAY, MAY 31 – All presentations will take place in the **Ballroom**

09:00	–	09:25	**Zohar Yosibash**, *Maxime Levy*, *Crack nucleation in a 1D heterogeneous bar: h- and p-FE approximation of a phase field model*	**Chair** Harri Hakula
09:25	–	09:50	**Nima Azizi**, Wolfgang Dornisch, *An effort to utilize high order exact geometrically defined Reissner-Mindlin spectral shell elements: Advantages and problems*	
09:50	–	10:15	Norbert Heuer, **Torsten Linß**, *Uniform convergence of an arbitrary order balanced FEM applied to a singularly perturbed shell problem*	
10:15	–	**10:40**	**Coffee Break**	
10:40	–	11:05	**Daniele Boffi**, *Model order reduction for parametric eigenvalue problems*	**Chair** Christoph Schwab
11:05	–	11:30	**Lukasz Kaczmarczyk**, Christophe-Alexandre Chalons-Mouriesse, Chris Pearce, *A mixed finite element method for 3D in-elasticity problems at large strains with weakly imposed symmetry*	

11:30	–	11:55	**Alexey Chernov**, Tung Le, *On analytic and Gevrey class regularity for parametric elliptic eigenvalue problems*	
11:55	–	12:20	**Sascha Eisenträger**, Wadhah Garhuom, Fabian Duvigneau, Stefan Löhnert, Alexander Düster, Dominik Schillinger, *On a stabilization technique for fictitious domain methods based on an eigenvalue decomposition: Time-dependent problems*	
12:20	–	**14:15**	**Lunch**	
14:15	–		**Wednesday afternoon EXCURSION –** Tour and dinner in Limassol (Guided tour in Limassol and dinner at “Folia tou Drakou” tavern in Pentakomo)	

THURSDAY, JUNE 1 – All presentations will take place in the room **Ballroom**

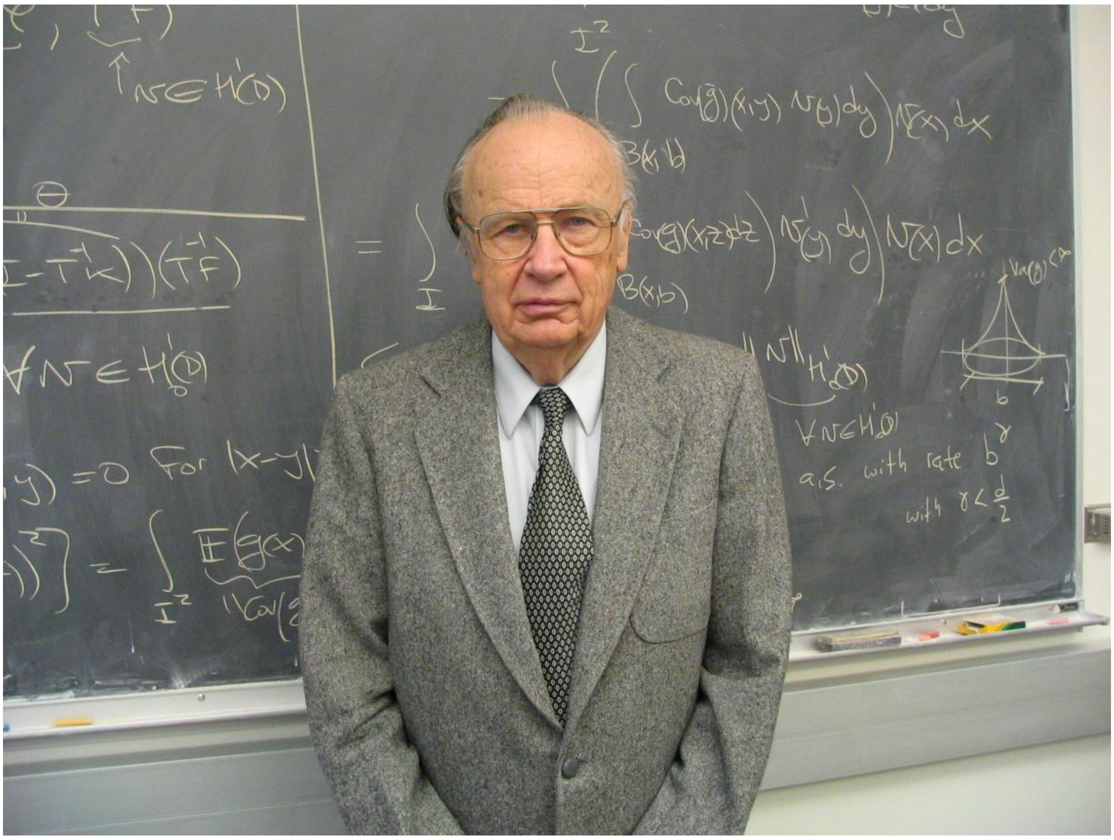
09:00	–	09:25	**Harri Hakula**, *Conformal mappings, reciprocal error estimates, and Laplace-Beltrami operator*	**Chair** Andreas Schröder
09:25	–	09:50	Bernard Kapidani, **Rafael Vázquez**, *Fast computation of electromagnetic wave propagation with spline differential forms*	
09:50	–	10:15	**Deepesh Toshniwal**, *Discrete de Rham complex of hierarchical spline differential forms in* $\boldsymbol{R}^n$	
10:15	–	**10:40**	**Coffee Break**	
10:40	–	11:05	**Stefan Tyoler**, Stefan Takacs, *Efficient computation of a spline basis for adaptive multipatch discretizations*	**Chair** Leszek Demkowicz
11:05	–	11:30	**Dohyun Kim**, Brendan Keith, *DynAMO: Dynamic anticipatory mesh optimization with reinforcement learning*	
11:30	–	11:55	**Espen Sande**, Michael Floater, Carla Manni, Hendrik Speleers, *Best approximations of matrices and differential operators*	
11:55	–	12:20	Erik Burman, **Guillaume Delay**, Alexandre Ern, *The unique continuation problem for the heat equation discretized with a high-order space-time nonconforming method*	
12:20	–	**12:30**	**END OF CONFERENCE**	

Monday, May 29

Talks

Leszek Demkowicz

Ivo Babuška tribute



Ivo M.Babuška (22 March 1926 – 12 April 2023)

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Full Envelope DPG Approximation for Electromagnetic Waveguides. Stability and Convergence Analysis

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Key Words: *DPG, Full envelope, stability*

The presented work started with a convergence and stability analysis for the so-called full envelope approximation used in analyzing optical amplifiers (lasers). The specific problem of interest was the simulation of *Transverse Mode Instabilities (TMI)* [1]. The problem translates into the solution of a system of two nonlinear time-harmonic Maxwell equations coupled with a transient heat equation. Simulation of a 1 m long fiber involves the resolution of 10 M wavelenghts. A superefficient MPI/openMP hp FE code run on a supercomputer gets you to the range of ten thousand wavelenghts. The resolution of the additional thousand wavelenghths is done using an exponential ansatz e^{ikz} in the z-coordinate. This results in a non-standard Maxwell problem.

The stability and convergence analysis for the problem has been restricted to the linear case only [3, 2]. It turns out that the modified Maxwell problem is stable if and only if the original waveguide problem is stable and the boundedness below stability constants are identical. We have converged to the task of determining the boundedness below constant.

The stability analysis started with an easier, acoustic waveguide problem. Separation of variables leads to an eigenproblem for a self-adjoint operator in the transverse plane (in x, y). Expansion of the solution in terms of the corresponding eigenvectors leads then to a decoupled system of ODEs, and a stability analysis for a two-point BVP for an ODE parametrized with the corresponding eigenvalues. The L^2-orthogonality of the eigenmodes and the stability result for a single mode, lead then to the final result: the boundedness below constant depends inversely linearly upon the length L of the waveguide.

The corresponding stability for the Maxwell waveguide turned out to be unexpectedly difficult. The equation is vector-valued so a direct separation of variables is out to begin with. An exponential ansatz in z leads to a non-standard eigenproblem involving an operator that is non-self adjoint even for the easiest, homogeneous case. The answer to the problem came from a tricky analysis of the eigenproblem combined with the perturbation technique for perturbed self-adjoint operators. The use of perturbation theory requires an assumption on the smallness of perturbation of the dielectric constant (around a constant value) but with no additional assumptions on its differentiability (discontinuities are allowed). In the end, the final result is similar to that for the acoustic waveguide - the boundedness below constant depends inversely linearly on L.

References

[1] S. Henneking, J. Grosek, and L. Demkowicz. Parallel simulations of high-power optical fiber amplifiers. *Lect. Notes Comput. Sci. Eng.*, 2022. accepted.

[2] M. Melenk, L. Demkowicz, S. Henneking, and J. Badger. Analysis of a non-homogeneous EM waveguide problem. Technical report, Oden Institute, The University of Texas at Austin, Austin, TX 78712, 2023. In preparation.

[3] M. Melenk, L. Demkowicz, S. Henneking, and J. Badger. Analysis of full envelope approximation of EM waveguide problems using DPG method. Technical report, The University of Texas at Austin, Austin, TX 78712, 2023. In preparation.

Multiscale FEM and DPG methodology for upscaling in solid mechanics

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Key Words: *Upscaling, DPG, elastic-visco-plastic material*

We present the Multiscale Finite Element Method (MsFEM) [3] enhanced for linear static and eigen problems as well as quasi-static inelastic problems with highly oscillating material coefficients and multi-field, higher-order FEM approximation stabilized by the DPG methodology. The concept of MsFEM is based on coarse and fine discretization. The coarse, macro-scale mesh discretizes the whole domain with a relatively small number of degrees of freedom, while an auxiliary fine mesh, generated for each coarse element, captures the micro-scale details of the material and is used for on-line computation of special (optimal) trial shape functions by the solution of local boundary value problems in each coarse element.

We focus here on the elastic-visco-plastic deformations of solids. Thus, we deal with a time-dependent problem for which also temporal upscaling is performed. Our enhancement uses static condensation, previously examined for 2D problems [2], that transmits the construction of the special trial functions exclusively to the coarse element interfaces. This way, both the approximation error and computational cost are reduced. Higher-order shape functions increase the efficiency of calculations, and the DPG methodology [1] provides the discrete stability for the mixed formulation due to constructed on-the-fly optimal test functions. Good accuracy and convergence of the improved method will be illustrated by solutions of selected 3D linear and nonlinear problems.

References

[1] L. Demkowicz and J. Gopalakrishnan. A class of discontinuous Petrov-Galerkin methods. Part I. The transport equation. *Computer Methods in Applied Mechanics and Engineering*, (Vol. 199):1558–1572, 2010.

[2] Marta Oleksy, Witold Cecot, Waldemar Rachowicz, and Michal Nessel. An improved Multiscale FEM for the free vibrations of heterogeneous solids. *Computers & Mathematics with Applications*, (Vol. 110):110–122, 2022.

[3] X. Wu T. Hou. A multiscale finite element method for elliptic problems in composite materials and porous media. *Journal of Computational Physics*, (Vol. 134):169–189, 1997.

Scalable hp-adaptive DPG multigrid solver with applications in high-frequency wave propagation

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Key Words: *hp-Adaptive Mesh Refinement, Multigrid, Wave Propagation*

Scalable solution of time-harmonic high-frequency wave propagation problemsincluding acoustic Helmholtz, elastic Helmholtz, and time-harmonic Maxwellremains a challenge in mathematics and scientific computing. One difficulty is that classical discretization techniques (e.g., Galerkin finite elements, finite difference, etc.) yield indefinite discrete systems that preclude use of many scalable solution algorithms. Significant progress has been made to develop specialized preconditioners for high-frequency wave propagation problems, but robust and scalable solvers for general problems including non-homogenous media and complex geometries remain elusive. An alternative approach is to use minimum residual discretizationsthat yield definite discrete systemsand may be more amenable preconditioning.

This work details a scalable multigrid solver for high-frequency wave propagation problems discretized with the discontinuous Petrov-Galerkin (DPG) finite element methodologya minimum residual method. The DPG multigrid (DPG-MG) solver [2] additionally leverages mesh-independent stability and a built-in error indicator to define a hierarchy of hp-adaptive meshes on which multilevel preconditioning is performed. General unstructured meshes with elements of all types, as well as isotropic and anisotropic hp-refinements, are supported.

We will first outline the scalable implementation of the DPG-MG solver. Efficacy of the solver will then be demonstrated via a number of three-dimensional wave propagation problems, including problems with more than 100 wavelengths, billions of degrees of freedom, non-homogeneous media, and complex geometries (e.g. tokamak). Finally, the DPG-MG solver employs a vertex-patch block smoother [1]; however, storing patch matrices can become prohibitive on high-order meshes. We thus conclude with a discussion of strategies to mitigate high-order patch storage, including use of mixed-precision and H-matrix compression.

References

[1] Douglas N Arnold, Richard S Falk, and Ragnar Winther. Multigrid in H (div) and H (curl). *Numerische Mathematik*, 85(2):197–217, 2000.

[2] Socratis Petrides and Leszek Demkowicz. An adaptive multigrid solver for DPG methods with applications in linear acoustics and electromagnetics. *Comput. Math. with Appl.*, 87:12–26, 2021.

The DPG method as a time-integration scheme for linear and non-linear transient PDEs

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Key Words: *DPG method, Ultraweak formulation, Optimal test functions, Multistage method, Exponential integrators, Semi-linear problems*

The main idea of the Discontinuous Petrov-Galerkin (DPG) method [2] is to select optimal test functions that realize the supremum in the inf-sup condition in order to guarantee discrete stability. Recently, we applied the DPG method only in the time variable in order to obtain stable DPG-based time-marching schemes for linear PDEs [3, 4]. In this work, we first introduce the method for transient linear problems, and then we explain how we employ this construction as a baseline for constructing new multistage methods for nonlinear transient PDEs. We semidiscretize the PDE in space by a classical Bubnov-Galerkin method. Then, we approximate the nonlinear term by a polynomial in time employing known values of the solution from previous stages. Considering an ultraweak variational formulation of the linearized problem we compute the optimal test functions analytically, which are exponential related functions. Finally, we obtain a time-marching scheme that locally computes the solution in the element interiors and performs post-processing for the trace variables. This variational construction allows to develop a-posteriori error estimations [1] for designing (goal-oriented) adaptive strategies.

References

[1] Judit Muñoz-Matute, Leszek Demkowicz, and David Pardo. Error representation of the time-marching DPG scheme. *Computer methods in applied mechanics and engineering*, 391:114480, 2022.

[2] Judit Muñoz-Matute, Leszek Demkowicz, and Nathan V Roberts. Combining DPG in space with DPG time-marching scheme for the transient advection–reaction equation. *Computer Methods in Applied Mechanics and Engineering*, 402:115471, 2022.

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The Entropic Finite Element Method

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Key Words: *High-order, pointwise positivity, structure-preserving discretization, Banach–Lie group, variational inequality*

One of the longest-standing challenges in finite element analysis is to develop a stable, high-order Galerkin method that strictly enforces *pointwise* bound constraints. The entropic finite element method (EFEM) is a nonlinear, structure-preserving method with these three properties. This talk will introduce EFEM and describe its capability for treating free-boundary problems, enforcing discrete maximum principles, and designing scalable, mesh-independent algorithms for inverse design problems.

Immersed space-time hp-finite elements for temperature evolution in laser powder bed fusion

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Key Words: *Laser powder bed fusion, part-scale simulation, space-time finite elements, hp-refinement, finite cell method*

Laser powder bed fusion (PBF-LB) is an additive manufacturing technology for directly printing three-dimensional metal structures. During the printing process, metal powder is added layer-by-layer, while being selectively melted by a high-power laser. Thermal simulations of PBF-LB can help to automate and optimize the printing process at a low computational cost, but even cheap thermal models based on classical time-stepping schemes and finite element discretizations in space struggle with the extreme differences in spatial and temporal scales.

We present a space-time finite element approach with local hp-refinement in four dimensions that can resolve the local characteristics of the temperature field also in time [1]. We use the superposition concept of the multi-level hp-method to build four-dimensional hp-bases using very simple algorithms and data structures [2]. Coarse elements marked for refinement are overlayed by the sixteen smaller elements resulting from bisecting it in the three spatial directions and in time. We subdivide the space-time problem into shorter time-slabs to obtain a sequence of smaller problems with reasonable sizes that we refine towards the laser path.

We apply this approach to several PBF-LB examples with multiple layers of metal powder, where we use the finite cell method to resolve the expanding domain and distinguish between powder and solidified metal. The additional quadrature cost of the finite cell method is comparably low in our space-time method as the element duration of most immersed elements is significantly longer compared to the fine elements around the laser spot. We discuss the modeling error, the numerical accuracy, and the computational performance of our approach.

[1] P. Kopp, V. Calo, E. Rank, S. Kollmannsberger, Space-time hp-finite elements for heat evolution in laser powder bed fusion additive manufacturing, Engineering with Computers, 38, 4879 – 4893 (2022). `https://doi.org/10.1007/s00366-022-01719-1`

[2] P. Kopp, E. Rank, V. Calo, S. Kollmannsberger, Efficient multi-level hp-finite elements in arbitrary dimensions. Computer Methods in Applied Mechanics and Engineering, 401, 115575 (2022). `https://doi.org/10.1016/j.cma.2022.115575`

Space-Time Continuous Galerkin Methods for the Wave Equation

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Key Words: *Wave Equation, Space-Time, Continuous Galerkin, CFL Condition*

For the discretisation of time-dependent partial differential equations, classical approaches are explicit or implicit time-stepping schemes together with finite element methods in space. An alternative approach is the usage of space-time methods, i.e., the temporal variable t is just another spatial variable. Thus, the space-time domain Q is discretised, and the resulting global linear system has to be solved at once.

In this talk, the model problem is the wave equation in second-order formulation. First, we recall a space-time variational formulation of the wave equation in $H^1(Q)$, applying integration by parts for the temporal and spatial variables, see [2, 3]. Second, space-time discretisation schemes, using piecewise polynomials, globally continuous ansatz and test functions, are considered. For a tensor-product approach, stability and related CFL conditions are discussed, see [1, 2, 3, 4]. In the last part of the talk, numerical examples for a one-dimensional spatial domain and a two-dimensional spatial domain are presented.

References

[1] Richard Löscher, Olaf Steinbach, and Marco Zank. Numerical results for an unconditionally stable space-time finite element method for the wave equation. In *Domain Decomposition Methods in Science and Engineering XXVI*, volume 145 of *Lect. Notes Comput. Sci. Eng.*, pages 587–594. Springer, Cham, 2023.

[2] Olaf Steinbach and Marco Zank. A stabilized space-time finite element method for the wave equation. In *Advanced finite element methods with applications*, volume 128 of *Lect. Notes Comput. Sci. Eng.*, pages 341–370. Springer, Cham, 2019.

[3] Marco Zank. *Inf-Sup Stable Space-Time Methods for Time-Dependent Partial Differential Equations*, volume 36 of *Monographic Series TU Graz: Computation in Engineering and Science*. 2020.

[4] Marco Zank. Higher-order space-time continuous Galerkin methods for the wave equation. *World Congress in Computational Mechanics and ECCOMAS Congress*, 700:1–10, 2021.

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A space–time continuous and coercive formulation for the wave equation

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Key Words: *Wave equation, space–time method, variational problem, coercivity, Lax–Milgram theorem, C^1-discretisation*

Space–time (XT) methods discretise the full space–time domain of definition of initial–boundary value problems (IBVP), contrary to the classical method of lines and Rothe's method. When the PDE to be approximated is the acoustic wave equation $\partial_t^2 u - c^2 \Delta u = f$, no stable XT variational formulation that accommodates general discrete spaces is available.

We use a Morawetz-multiplier technique (i.e. integration by parts with well-chosen test functions, coming from scattering theory) to derive an XT variational formulation that is continuous and coercive (sign-definite) in a norm stronger than $H^1(\Omega \times (0, T))$. This applies to IBVPs for the wave equation with constant wavespeed c, impedance boundary conditions $\partial_t u + \theta c \, \partial_{\mathbf{n}} u = g$ on the boundary of a star-shaped space domain, and, possibly, a star-shaped Dirichlet scatterer. The continuity and the coercivity constants are simple explicit expressions of the IBVP parameters. Lax–Milgram theorem and Céa lemma allow to discretise the proposed variational formulation with any discrete space $V_h \subset C^1(\overline{\Omega \times (0, T)})$ in a stable way and derive error bounds. We show some numerical experiments that confirm the theoretical results.

Modeling, Calibration, and Validation of Powder Bed Fusion Process Simulations using the Finite Cell Method

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Key Words: *Powder bed fusion, Finite Cell Method, Additive Manufacturing, Experimental Validation*

The extreme scale ranges in both space and time involved in powder bed fusion process (PBF) of metals as well as the geometrical complexity of the parts produced by means of PBF technology call for flexible numerical approaches. Therefore, immersed boundary methods seem to offer a valid alternative to the traditional mesh-conforming finite element method. A well established immersed methodology is the so-called Finite Cell Method (FCM), which has already been applied successfully in several contexts, from structural analysis to biomedical applications since it allows to easily deal with complex shape components, otherwise non trivial or even impossible to mesh in a conform manner.

In the present contribution, we present the application of FCM in the context of high-fidelity thermal [1] and thermomechanical [2] analyses of PBF processes. The proposed numerical scheme is first calibrated and then validated with respect to several experimental measurements. The developed thermal model is then applied to investigate the influence of process-induced material discontinuities on the melt pool morphology whereas the thermomechanical analysis is used to predict residual stresses in the solidified material due to the rapid melting-solidification cycle occurring in PBF processes.

The ability of FCM to capture complex geometries in an implicit manner allows a straightforward solution even when amorphous material discontinuities are present beneath the powder layer due to process-induced defects (e.g., balling, keyhole porosity, lack-of-fusion defects) thus providing a valuable tool to investigate the influence of such a kind of defects on the melt pool morphology.

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Adaptive phase-field simulations with the parallel finite cell method

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Key Words: *Phase-field fracture, finite cell method, multi-level hp-refinement, parallel computations*

The numerical simulation of crack propagation in the context of phase-field models is challenging for two main reasons. First, the regularization of the sharp crack is based on a length-scale parameter that necessitates extremely fine meshes in the vicinity of the crack. Secondly, a staggered solution scheme based on decoupling of the phase-field and mechanical equation is typically used which suffers from slow convergence.

To reduce the computational effort and enable the solution of large-scale problems, we present a numerical framework based on a combination of a phase-field model for brittle fracture with the Finite Cell Method (FCM), multi-level hp-refinement, and parallel computing. Integrating the FCM [1] as an embedded domain approach enables the efficient simulation of complex geometries without the need to generate boundary-conforming meshes. Multi-level hp-refinement allows for a locally refined mesh that dynamically adapts to the crack path. As presented in [3], implementing the two discretization techniques in an MPI-parallel setting enables the efficient numerical solution of complex geometrical and physical problems.

In the present contribution, we extend the parallel framework to the simulation of crack propagation by combining it with a phase-field model for brittle fracture [2]. The potential of the proposed numerical framework will be investigated based on several 3-dimensional practical examples.

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hp-Version Polytopic Discontinuous Galerkin Methods for Radiation Transport Problems

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Key Words: *hp-finite element methods; discontinuous Galerkin methods; linear Boltzmann transport problem; polytopic meshes*

In this talk we develop hp–version discontinuous Galerkin finite element methods (DGFEMs) for the discretization of the radiation transport problem on general (spatial) computational meshes consisting of polygonal/polyhedral (polytopic) elements. Our particular interest is the application to medical treatment planning in clinical radiotherapy. Here we study both the stability and a priori error analysis of the proposed scheme. The implementation is based on exploiting a nodal approximation in energy and angle, together with fast numerical integration techniques on the spatial polytopic mesh; this approach leads to a highly parallelisable algorithm whereby a large number of linear transport solves must be computed. Numerical experiments are presented to highlight the accuracy of the proposed method, as well as to benchmark with more standard kinetic Monte Carlo simulations.

A hybridizable discontinuous Galerkin method with characteristic variables for high-frequency wave propagtion problems

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Key Words: *hybridizable discontinuous Galerkin, domain decomposition, iterative solver, high-frequency wave propagation*

I will present a new hybridization procedure for discontinuous Galerkin (DG) discretizations of high-frequency Helmholtz problems. In contrast to standard hybrizable discontinuous Galerkin (HDG) schemes that employ a Dirichlet trace as a Lagrange multiplier [3], this new approach called CHDG utilizes characteristic variables [7].

Although DG, HDG and CHDG produce exactly the same numerical solution, the new choice of auxiliary unknown changes the properties of the reduced system, which exhibit a structure similar to optimized Schwartz domain decomposition methods (see e.g. [2, 4]) and ultra weak Trefftz formulations (see [1, 5, 6], for instance). In particular, I will show that a simple fixed point iteration always converges to solve the CHDG reduced linear system, which is not the case of for standard DG and HDG schemes.

I will also numerically compare different iterative procedures including fixed point, GMRES and CGN iterations on increasingly complex benchmarks. We will see that on all the considered test cases, the number of iterations is reduced for CHDG as compared to DG and HDG, often by a large amount. Interestingly, the CGN iteration (which is often disregarded) converges as fast as the GMRES iteration without restart for the CHDG system.

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Tuesday, May 30

Talks

Isogeometric analysis: advances and applications with a special focus on dynamic problems

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Key Words: *Isogeometric Analysis, Structural Dynamics, Transient Problems*

Isogeometric Analysis (IGA) is a successful simulation framework originally proposed by T.J.R. Hughes et al., in 2005, with the aim of bridging Computational Mechanics and Computer Aided Design. In addition to this, thanks to the high-regularity properties of its basis functions, IGA has shown a better accuracy per degree-of-freedom and an enhanced robustness with respect to standard finite elements in many applications - ranging from solids and structures to fluids, as well as to different kinds of coupled problems - opening also the door for the approximation in primal form of higher-order partial differential equations. In particular, the above-mentioned higher-regularity properties of IGA make it particularly attractive for the efficient and accurate simulation of structural dynamics and transient problems. In this lecture, after a concise introduction on the basic concepts of isogeometric analysis and its potential advantages, some IGA recent advances in the context of structural dynamics and transient problems will be presented, along with an overview of interesting applications from different fields of Engineering.

Low-rank solver for Isogeometric Analysis

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Key Words: *Isogeometric Analysis, Low-rank, solver*

Similarly to other numerical methods to solve PDEs, Isogeometric Analysis suffers from the so-called curse of dimensionality, i.e. memory storage and computational effort grow exponentially with respect to the problem's dimension. In this talk we propose low-rank techniques that can overcome those issues. A low-rank decomposition of the linear system matrix kernel is combined with a new suited iterative solver. In particular, the non-tensor product coefficients is approximated with the sum of few Kronecker-product functions, and thus the linear system matrix results in the sum of few Kronecker-product matrices. This yields a small memory footprint and cost for matrix products. The techniques to approximate the linear system matrix in low-rank format are already present in literature. The novelty of our work is the development of a specialized iterative solver combined with a preconditioning strategy. Truncations and compressions of the tensors are employed to keep the rank of the iterates low. The preconditioner is based on the Fast Diagonalization method, applied in the isogeometric context in [1], that is recast to be compatible to the chosen tensor format. Our goal is to compute the solution of the problem in $O(n)$ FLOPs, where n is the number of univariate degrees of freedom. We also show some numerical experiments.

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Fast Poisson solvers for Isogeometric Analysis

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Key Words: *Isogeometric Analysis, tensor product, Fast Fourier Transform*

Consider the Poisson problem on a d-dimensional cube. It is well-known that, if the problem is discretized with linear finite elements on a uniform tensor product mesh, the resulting stiffness matrix can be diagonalized using the Fast Fourier Transform. This fact can be exploited to solve the linear system yielding $O(N \log N)$ complexity, where N represents the number of degrees of freedom. Such approach is referred to as a fast Poisson solver.

In this talk, we show how to generalize this idea to the case of B-splines of arbitrary degree p. The resulting algorithm solves the linear system with $O((N + p) \log N)$ complexity. This is achieved by first splitting the spline space into an outlier-free subspace and a subspace with low dimension. On the latter subspace, the eigenvectors of the problem are computed numerically. On the former subspace, on the other hand, the eigenvectors are approximated using interpolated sinusoidal functions. The resulting approximated eigendecomposition can be used as a preconditioner for the linear system, yielding extremely fast convergence independently of N and p.

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Inexpensive polynomial-degree-robust equilibrated flux a posteriori estimates for isogeometric analysis

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Key Words: *Isogeometric Analysis, High Order, A Posteriori Error Estimate, Equilibrated Flux, Partition of Unity, Inexpensive Equilibration, Robustness, Hierarchical Splines*

We consider isogeometric discretizations of the Poisson model problem, focusing on high polynomial degrees and strong hierarchical refinements. We derive a posteriori error estimates by equilibrated fluxes, i.e., vector-valued mapped piecewise polynomials lying in the $\mathbf{H}(\mathrm{div})$ space which appropriately approximate the desired divergence constraint. Our estimates are constant-free in the leading term, locally efficient, and robust with respect to the polynomial degree. They are also robust with respect to the number of hanging nodes arising in adaptive mesh refinement employing hierarchical B-splines. Two partitions of unity are designed, one with larger supports corresponding to the mapped splines, and one with small supports corresponding to mapped piecewise affine polynomials. The equilibration is only performed on the small supports, avoiding the higher computational price of equilibration on the large supports or even a global system solve. Thus, the derived estimates are also as inexpensive as possible. An abstract framework for such a setting, extending that of [1] and the references therein, is developed. Its application to a specific situation only requests a verification of a few clearly identified assumptions. Numerical experiments illustrate the theoretical developments, cf. Figure 1. All the details are provided in [2].

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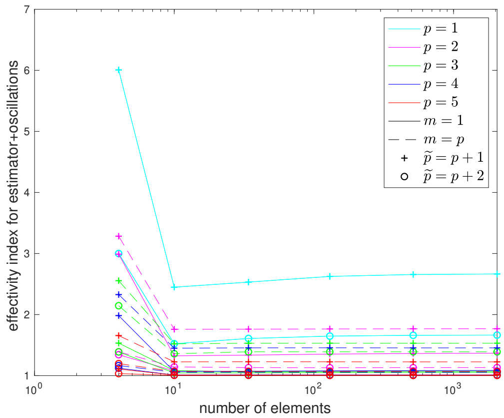

Figure 1: Effectivity indices (estimate/error) with an artificial refinement enforcing a high number of hanging nodes, polynomial degrees $p \in \{1, \ldots, 5\}$, multiplicities $m \in \{1, p\}$, equilibration polynomial degrees $\widetilde{p} \in \{p + 1, p + 2\}$.

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Anisotropic refinement with LR-splines

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Key Words: LR-spline, anisotropic, refinement

There are many techniques to achieve local refinement in the context of locally tensor product polynomial spline spaces. Among them a minority allow for anisotropic refinement for instance LR-splines [3] but without further assumptions the LR generating set can be linearly dependent. I will describe a construction, based on [1] and [2] that guarantees local linear independence of the generating set and a desired resolution.

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IgANets: Physics-Informed Machine Learning Embedded Into Isogeometric Analysis

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Key Words: *Physics-Informed Machine Learning, Isogeometric Analysis*

Many engineering problems of practical interest are modelled by systems of partial differential equations equipped with initial and boundary conditions and complemented by problem-specific constitutive laws. For decades, numerical methods like the finite element method have been the method of choice for computing approximate solutions to problems that cannot be solved analytically. Starting with the seminal paper [5] on physics-informed neural networks (PINNs), a new paradigm has entered the stage: learning the behavior of the problem instead of discretizing it and solving the resulting systems of equations brute-force. Next to PINNs, several alternative approaches like DeepONets [2] and Fourier neural networks [1] have been proposed in recent years. Their ease of implementation and fast response time, once training is completed, makes learning-based methods particularly attractive for engineering applications as they offer the opportunity to explore many different designs without costly simulation.

In this talk we propose a novel approach – *IgANets* – to embed the physics-informed machine learning paradigm into the framework of Isogeometric Analysis (IGA) to combine the best of both worlds. IGA is an extension of the finite element method that integrates the simulation-based analysis into the computer-aided geometric design pipeline. In short, the same mathematical formalism, namely (adaptive) B-splines or NURBS, that is used to model the geometry is adopted to represent the approximate solution, which is computed following the same strategy as in classical finite elements. In contrast to classical PINNs [5], which predict point-wise solution values to (initial-)boundary-value problems directly, our IgANets [4] learn solutions in terms of their expansion coefficients relative to a given B-Spline or NURBS basis. This approach is furthermore used to encode the geometry and other problem parameters such as boundary conditions and parameters of the constitutive laws and feed them into the feed-forward neural network as inputs. Once trained, our IgANets make it possible to explorer various designs from a family of similar problem configurations efficiently without the need to perform a computationally expensive simulation for each new problem configuration.

Next to discussing the IgANets' underlying concepts and presenting numerical results, we will shed some light on the technical details of our C++ reference implementation in Torch. In particular, we will discuss a hardware-optimized implementation of B-splines based on the matrix form representation from [3] that is particularly suited for an efficient backpropagation step. We contrast the computational costs of our approach with that of classical PINNs.

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Space-time IGA

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Key Words: *isogeometric analysis, splines, space-time*

High-degree and continuity splines (or NURBS, etc.) bring to isogeometric analysis (see [1]) high accuracy per degree-of-freedom but also pose significant challenges at the computational level: using standard finite element routines, the computational cost grows too fast with respect to the degree, making degree raising excessively expensive. This problem is even more relevant in space-time isogeometric discretizations, that is, when adopting smooth spline discretization in space and time. We proposed in [2] a class of solvers that exploits the tensor construction of spline spaces and achieves high efficiency thanks to linear algebra tensor methods ([2]). I will then discuss the use, advantages and disadvantages, of space-time isogeometric analysis for parabolic equations and beyond.

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Exponential convergence of hp FEM for spectral fractional diffusion in polygons

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Key Words: *Fractional diffusion, Nonlocal operators, Dunford-Taylor calculus, Anisotropic hprefinement, Geometric corner mesh refinement, Exponential convergence, n-width*

We report recent results from [2, 3] and [1]. For the spectral fractional diffusion operator L^s of order $2s$, $s \in (0, 1)$, in bounded, curvilinear polygonal domains $\Omega \subset \mathbb{R}^2$, we prove exponential convergence of two classes of hp FE discretizations under the assumption of analytic data (coefficients and source terms, without any boundary compatibility), in the natural fractional Sobolev norm $\mathbb{H}^s(\Omega)$. The first hp discretization is based on writing the solution as a co-normal derivative of a $2 + 1$-dimensional local, linear elliptic boundary value problem, going back to Caffarelli, Sylvestre and Stinga. To this degenerate, local 2nd order divergence-form PDE an hp-FE discretization with exponential convergence from [1] is applied. A diagonalization in the extended variable reduces the numerical approximation of the inverse of the spectral fractional diffusion operator to the numerical approximation of a system of local, decoupled, second order reaction-diffusion equations in Ω.

Leveraging results on robust exponential convergence of hp-FEM for second order, linear reaction diffusion boundary value problems in Ω, exponential convergence rates for solutions $u \in \mathbb{H}^s(\Omega)$ of $L^s u = f$ follow. Key ingredient in this hp-FEM are boundary fitted meshes with geometric mesh refinement towards $\partial\Omega$.

The second discretization is based on exponentially convergent numerical sinc quadrature approximations of the Balakrishnan integral representation of L^{-s} combined with hp-FE discretizations of a decoupled system of local, linear, singularly perturbed reaction-diffusion equations in Ω. The present analysis for either approach extends to (polygonal subsets $\tilde{\mathcal{M}}$ of) analytic, compact 2-manifolds $\mathcal{M}$, parametrized by a global, analytic chart χ with polygonal Euclidean parameter domain $\Omega \subset \mathbb{R}^2$.

Numerical experiments with the code NGSOLVE of J. Schöberl (Vienna) [9, 10] for model problems in nonconvex polygonal domains and with incompatible data confirm the theoretical results.

Exponentially small bounds on Kolmogorov n-widths of solution sets for spectral fractional diffusion in curvilinear polygons and for analytic source terms are deduced, which imply exponential convergence of MOR and RB approaches as in [4, 5].

Related work includes an hp-FE analysis of the *integral fractional Laplacean* [8, 7, 6] presented by J.M. Melenk and C. Marcati.

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Weighted analytic regularity for the integral fractional Laplacian in polygons

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Key Words: *fractional Laplacian, analytic regularity, corner domains, weighted Sobolev spaces*

The solutions to the fractional Laplace problem are well-known to have low regularity in classical spaces, even when the domain and the right-hand side are smooth. In particular, the solutions are smooth in the interior of the domain, but lose regularity at its boundary.

In this talk, I will consider the integral fractional Laplace problem in polygonal domains, with analytic right-hand sides. In this case, the solutions have both corner and edge singularities. By analyzing their regularity in corner- and edge-weighted Sobolev spaces, we are able to recover analytic-type estimates [2]. The proof relies on the analysis of the (3-dimensional) Caffarelli-Silvestre extension of the problem; the result also paves the way for the design of exponentially convergent numerical schemes [1].

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exponential convergence of hp-FEM for the integral fractional Laplacian

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Key Words: *hp-FEM, exponential convergence, fractional diffusion, anisotropic elements*

For the Dirichlet problem of the integral fractional Laplacian in a polygon Ω and analytic right-hand side, we show exponential convergence of the hp-FEM based on suitably designed meshes, [2]. These meshes are geometrically refined towards the edges and corners of Ω. The geometric refinement towards the edges results in anisotropic meshes away from corners. The use of such anisotropic elements is crucial for the exponential convergence result. These mesh design principles are the same ones as those for hp-FEM discretizations of the Dirichlet spectral fractional Laplacian in polygons, for which [1] recently established exponential convergence.

The hp-FEM convergence result relies on the recent [3], where weighted analytic regularity of the solution is shown in a way that captures both the analyticity of the solution in Ω and the singular behavior near the boundary. Near the boundary the solution has an anisotropic behavior: near edges but away from the corners, the solution is smooth in tangential direction and higher order derivatives in normal direction are singular at edges. At the corners, also higher order tangential derivatives are singular. This behavior is captured in terms of weights that are products of powers of the distances from edges and corners.

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hp-finite elements with higher differentiability on meshes with hanging nodes

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Key Words: *hp-fem, differentiable basis functions, hanging nodes*

The construction of basis functions is an important issue in the implementation of hp-finite element methods. In these methods the mesh size h as well as the polynomial degree p are adapted to the requirements of the (unknown) solution of the given problem. Due to their simplicity, well-known approximation properties, and several advantages with respect to the assembling process, tensor-product basis functions on quadrangles in 2D or hexahedrons in 3D are often used to construct such hp-basis functions, which are typically continuous as this (C^0-)differentiability property is needed in many conforming finite element discretizations.

In this talk the construction of hp-basis functions with higher C^k-differentiability is discussed. The basis functions are based on tensor-products and are defined on paraxial d-dimensional rectangular meshes with arbitrary hanging nodes and varying polynomial degree distributions. Hermite shape functions are combined with Gegenbauer polynomials to ensure that the support of the basis functions is independent of the prescribed differentiability. This, in particular, enables an efficient recursive computation of the constraints coefficients in the application of constrained approximation for hanging nodes [1]. It is emphasized that the approach allows for differentiability properties which can locally vary on the finite element mesh. The applicability of the approach is illustrated by several numerical examples.

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Larnaca, Cyprus

High order hierarchical spline methods on multi-patch geometries

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Key Words: *Isogeometric Analysis, Adaptivity, C^1 continuity, Multi-patch domains*

We propose an adaptive isogeometric method for the numerical approximation of (high order) partial differential equations defined on multi-patch geometries. By focusing on C^1 hierarchical spline constructions [1, 2], we will present a refinement algorithm with linear complexity which guarantees the construction of suitably graded hierarchical meshes that fulfill the condition for linear independence of the hierarchical basis. A selection of numerical examples will confirm the potential of the adaptive scheme on different multi-patch configurations.

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Improving the robustness of the finite cell method for nonlinear problems of solid mechanics

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Key Words: *Finite Cell Method, Eigenvalue Stabilization, Moment Fitting*

Fictitious domain methods are attractive discretization schemes since they facilitate the meshing process of problems with complicated geometry. The finite cell method (FCM) [3] represents a combination of the fictitious domain concept with high order finite elements. Due to the fact that the Cartesian meshes used in the FCM do not conform to the geometry of the underlying problem, several challenges are introduced deteriorating the robustness of the FCM especially for nonlinear problems. We will present and discuss different approaches of how to improve the robustness of the FCM. The influence of the numerical integration [1] and different stabilization techniques [2] applied to cut cells will be addressed and investigated for problems including large elastic and elastoplastic deformations.

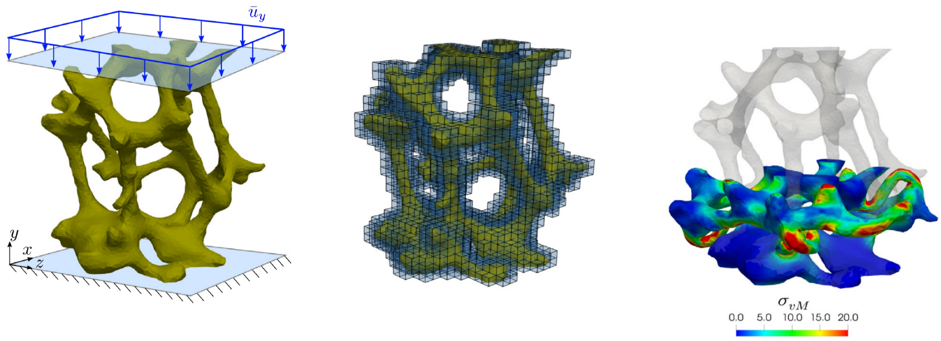

Figure 1: Single pore of a foam, finite cell grid, and deformed configuration.

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Larnaca, Cyprus

Deep neural networks for smooth approximation of physics with higher order and continuity basis functions

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Key Words: *Deep Neural Networks, Higher-order and continuity, Finite element method*

The PINN [3] introduced approximation of solutions of PDEs by DNN, $u(x) = DNN(x) = A_n\sigma\left(...\sigma(A_1x + B_1)...\right) + B_n$, where A_i are matrices, B_i bias vectors of DNN layers. To train the DNN a solution of PDE, the loss functions consider the residual of a PDE, e.g., $LOSS_{PDE}(x) = (-\epsilon\frac{d^2DNN(x)(x)}{dx^2} + \frac{dDNN(x)(x)}{dx} - 1.0)^2$, and b.c. $LOSS_{BC0}(0) = (-\epsilon\frac{dDNN(0)}{dx} + DNN(0) - 1.0)^2$, $LOSS_{BC1}(1) = (DNN(1))^2$. The training involves probing the loss at random points. The VPINN [2] introduces the weak PDE residuals, e.g., $LOSS_{weak}(v) = \left(\int_0^1 \left(\epsilon\frac{dDNN(x)}{dx}\frac{dv}{dx} + \frac{dDNN(x)}{dx}v\right)dx + DNN(0)v(0) - \int_0^1 DNN(x)dx + v(0)\right)^2$, and training involves probing with random test functions v. The DNN can also learn the solution of parametric PDE "at once". This requires a redefinition of PINN/VPINN loss functions, e.g., $LOSS_{PDE}(x,\epsilon)$ or $LOSS_{weak}(v,\epsilon)$ and $LOSS_{BC0}(0,\epsilon)$, and including random ϵ into the training process. In this talk, we discuss introducing the higher-order and continuity B-splines into this setup [1]. We consider approximating the solution of the parametric PDE with DNN. Alternatively, we approximate the solution of the parametric PDE with a combination of B-splines $u_h(x) = \sum_{i=1,...,N} u_iB_{i,p}(x)$. We train DNN the coefficients of the linear combination $u_i(\epsilon) = NN_i(\epsilon)$. We employ either PINN trained at points or VPINN trained with B-splines.
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Wednesday, May 31

Talks

Crack nucleation in a 1D heterogeneous bar: h- and p-FE approximation of a phase field model

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Key Words: *PFM, AT1, p-FEA, Heterogeneous*

In an attempt to accurately predict crack nucleation in human long bones, the AT1 phase field method (PFM) *for heterogeneous elastic materials* is considered following preliminary encouraging attempts [2, 3]. The AT1-PFM is investigated for a 1-D heterogeneous bar, formulated as a constrained minimization of a coupled weak-problem and solved by h- and p-FEM. For verification purposes, we present explicit analytical solutions for a 1-D bar incorporating linear, parabolic, and exponential $E(x)$ and $G_{Ic}(x)$ profiles [4].

We enforce the damage positivity in the weak formulation by penalization [1], and provide optimal penalization coefficients depending on the discretization and material heterogeneity for both h- and p-FEA. We also provide correction to the numerical values of G_{Ic} for the heterogeneous material profiles.

Numerical examples are provided to demonstrate the convergence rates obtained by h- and p-FEA.

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An effort to utilize high order exact geometrically defined Reissner-Mindlin spectral shell elements: Advantages and problems

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Key Words: *Spectral Element Method, NURBS, Reissner-Mindlin Shell*

A curved non-isoparametric Reissner-Mindlin shell element is developed for analyzing shell structures. To calculate the director vector accurately, similar to isogeometric analysis (IGA), the geometry is defined by utilization of the non-uniform rational B-splines (NURBS) imported directly from CAD files. Then, shape functions of the Legendre spectral element method (SEM) are used to interpolate the displacements. Consequently, the shell director vector and Jacobian of the transformation are calculated properly according to the presented formulation. On the other hand, in the Legendre SEM combined with the Gauss-Lobatto-Legendre quadrature, the integration points and the element nodes coincide. Thus, the calculation of interpolated director vector at integration points is not necessary. This is the source of either complexity or error in the calculation of proper local nodal systems in IGA shells [1]. Given the condition number of the stiffness matrix in the developed method, super high-order elements can also be used. The validity and convergence rate of the method in small deformation analysis are investigated and verified through various cases of *h*- and *p*-refinement in challenging obstacle course problems. On the other hand, the above-mentioned assumptions utilized for interpolation of coordinates and displacements in the presented non-isoparametric element is the source of a problem in large deformation analysis. The problem, which stems from the rigid body rotation, is elaborated in more details both analytically and numerically and we will explain our effort to overcome this issue which occurs in the contribution.

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Uniform convergence of an arbitrary order balanced FEM applied to a singularly perturbed shell problem

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Key Words: *axisymmetrically loaded thin shell, singular perturbation, balanced norm, uniform convergence, conforming FEM, layer-adapted mesh*

We study a boundary value problem that describes the bending of an axisymmetrically loaded thin shell. The thickness of the shell appears in the differential equation as a singular perturbation parameter. As a consequence layers form and must be resolved by the numerical scheme.

We show that the energy norm naturally associated with the standard weak formulation fails to capture the layers. Using an idea by Lin and Stynes [1], we devise an alternative variational formulation whos induced norm (a so called "balanced norm") is stronger. This is then discretised using arbitrary order conforming FEM. We proof convergence on a layer-adapted mesh that is robust with respect to the perturbation parameter.

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Model order reduction for parametric eigenvalue problems

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Key Words: *Reduced order methods, Parametric eigenvalue problems, Finite element method*

Reduced order methods are consolidated and effective tools for the numerical approximation of parametric partial differential equations. The application of reduced models to eigenvalue problems is well understood in some very particular cases [5, 6].

The aim of our investigation is to discuss more critical cases and to show how to deal with cluster of eigenvalues, possibly leading to degenerate situations and crossings [3, 1, 2, 4].

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HOFEIM 2023
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A mixed Finite Element Method for 3D in-elasticity problems at large strains with weakly imposed symmetry

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Key Words: *Large strains, weakly imposed symmetry*

I present an extension of the mixed finite element for small strain elasticity [**?**, **?**] to large strain problems. The finite element formulation includes four independently approximated fields, i.e. stresses, logarithm stretches, rotation vectors, and displacements. The first two are associated with the conservation of linear momentum and conservation of angular momentum, respectively. The other two fields are associated with the constitutive equation and the consistency condition between displacements and deformation. The relationship between the rotation vectors and rotation tensor is established by an exponential map. The stresses are approximated in H(div) space, and the remaining three fields are in L2 space. This formulation creates a very sparse system of equations that is simple to parallelise, thereby enabling highly-scalable and robust solvers. The FE formulation is implemented in open-source software, MoFEM [**?**], developed by the GCEC.

This FE technology enables us to tackle problems with nearly incompressible soft elasto-plastic materials. Further, this new approach opens up the possibility of tackling robust problems in DD-driven approaches for large strains and multi-field formulations for computational plasticity and efficient error estimators for p-adaptivity.

On analytic and Gevrey class regularity for parametric elliptic eigenvalue problems

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Key Words: *Eigenvalue problems, parametric regularity, analytic and Gevrey-class functions*

We investigate a class of parametric elliptic eigenvalue problems where the coefficients (and hence the solution) may depend on a parameter y.

$$\begin{aligned} -\nabla \cdot (a(x,y)\nabla u(x,y)) + b(x,y)u(x,y) &= \lambda(y)c(x,y)u(x,y) \qquad && (x,y) \in D \times U, \\ u(x,y) &= 0 && (x,y) \in \partial D \times U, \end{aligned}$$

where the derivative operator ∇ acts in the physical variable $x \in D$, where D is a bounded Lipschitz domain in $\mathbb{R}^d$. The vector of parameters $y = (y_1, y_2, \dots) \in U$ has either finitely many or countably many components. Understanding the regularity of the solution as a function of y is important for construction of efficient numerical approximation schemes. Several approaches are available in the existing literature, e.g. the complex-analytic argument by Andreev and Schwab [1] and the real-variable argument by Gilbert et al. [2, 3]. The latter proof strategy is more explicit, but, due to the nonlinear nature of the problem, leads to slightly suboptimal results. In this talk we close this gap and (as a by-product) extend the analysis to the more general class of coefficients.

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HOFEIM 2023
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Larnaca, Cyprus

On a stabilization technique for fictitious domain methods based on an eigenvalue decomposition: Time-dependent problems

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Key Words: *Stabilization technique, Eigenvalue decomposition, Fictitious domain methods, Finite cell method, Spectral cell method, Structural dynamics.*

Fictitious domain methods are widely used in computational mechanics to solve problems involving complex geometries. However, when elements are poorly cut, ill-conditioning and stability problems can arise, which can compromise the accuracy of numerical simulations. In addition, in explicit dynamics a significant reduction in the critical time step size can be caused, depending on the smallest volume fraction of a cut element. To address these challenges, an eigenvalue stabilization technique is developed [1], which not only improves the condition number of the system matrices, but also increases the maximum time increment. Hence, making it possible to analyze time-dependent problems with greater accuracy and efficiency. To this end, the proposed approach identifies mode shapes with little support in the physical domain on an element level, which is easily achieved using the computed eigenvalues of the elemental matrices [2, 3] adding a negligible numerical overhead. A stabilization term is derived and added to both sides of the linear system of equations to ensure that the solution is not modified. On the one hand, the system matrices—$\mathbf{K}$ and $\mathbf{M}$—are stabilized and on the other hand, the right-hand side of the system of equations, i.e., the external load vector, is adjusted by a force correction term. The performance of the proposed technique is demonstrated through selected benchmark examples in a dynamic setting. Overall, this method provides a robust and efficient approach for solving problems using fictitious domain methods, and has potential applications in a wide range of engineering fields.

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Thursday, June 1

Talks

HOFEIM 2023
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Conformal Mappings, Reciprocal Error Estimates, and Laplace-Beltrami Operator

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Key Words: *Laplace-Beltrami, conformal mapping, p-version*

The conjugate function method for computing conformal mappings in the plane, see [2], can be extended to surfaces simply by replacing the standard Laplacian with the Laplace-Beltrami operator. In the Figure 1 a grid on a hemisphere is shown. In the context of high-order FEM the Laplace-Beltrami has previsously been discussed by Cantwell et al [1]. Remarkably, the

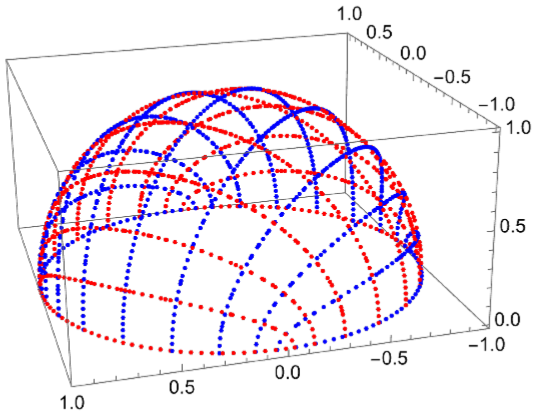

Figure 1: Schwarzian hemisphere. Each intersection of the gridlines is at right angles.

reciprocal error estimator is also equally valid on surfaces. Exponential convergence of the p-version is demonstrated over a series of numerical experiments. The reciprocal error estimator is compared with the standard auxiliary space error estimator.

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Fast computation of electromagnetic wave propagation with spline differential forms

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Key Words: *Maxwell equations, Hodge-$\star$ operators, Kronecker product, Splines*

We present a new structure-preserving numerical method which exhibits high order convergence and, contrarily to other high order geometric methods, does not rely on the geometric realization of any dual mesh. We use B-spline based de Rham complexes to construct two exact sequences of discrete differential forms: the primal sequence starts from the space of tensor-product splines of degree p and at least C^1 continuity. Similarly, the dual sequence starts from the space of tensor-product splines of degree $p-1$, which in the parametric domain coincides with the last space of the primal sequence.

The differential operators (gradient, curl and divergence) are condensed in the exterior derivative operator, and due to the high continuity of splines they are well defined both for the primal and the dual sequence. The method is completed with two sets of discrete Hodge-star operators, that relate the spaces of the two sequences, mapping the space of primal k-forms into the space of dual $(n-k)$-forms, and vice versa. The discrete Hodge-star operators encapsulate all the metric-dependent properties, including material properties, see [2].

We show a particular choice of the discrete Hodge-star operators inspired by [1] and how to compute them through the fast inversion of Kronecker product matrices. We apply the method to the solution of Maxwell equations [3], and show that it exhibits high order convergence and energy conservation, with computational times much lower than for standard Galerkin discretizations. We will also present preliminary results about the extension of the method to multi-patch geometries.

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Discrete de Rham complex of hierarchical spline differential forms in $\mathbb{R}^n$

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Key Words: *discrete de Rham complex, hierarchical B-splines, finite element exterior calculus*

Finite element exterior calculus (FEEC) is a framework for designing stable and accurate finite element discretizations for a wide variety of systems of PDEs. The involved finite element spaces are constructed using piecewise polynomial differential forms, and stability of the discrete problems is established by preserving at the discrete level the geometric, topological, algebraic and analytic structures that ensure well-posedness of the continuous problem. The framework achieves this using methods from differential geometry, algebraic topology, homological algebra and functional analysis. In this talk I will discuss the use of smooth splines within FEEC, they are the de facto standard for representing geometries of interest in engineering and offer superior accuracy in numerical simulations (per degree of freedom) compared to classical finite elements. In particular, for the hierarchical B-spline complex of discrete differential forms [1] on a domain $\Omega \subset \mathbb{R}^n$, I will present sufficient and locally verifiable conditions on the mesh refinement that guarantee the complex's exactness [2].

Joint work with: Kendrick Shepherd (BYU) and Rafael Vazquez (EPFL)

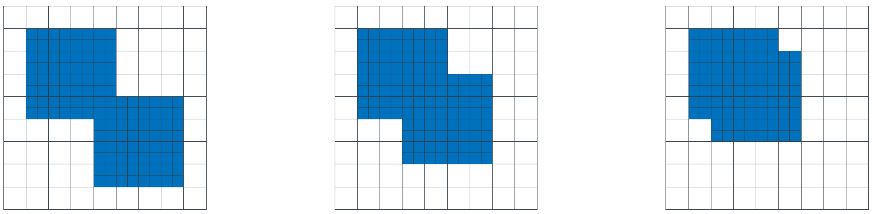

Figure 1: Only the left and right refinements lead to exact hierarchical B-spline complexes; the former is covered by the sufficient conditions from [2] and the latter by those from [1].

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Efficient computation of a spline basis for adaptive multipatch discretizations

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Key Words: *Isogeometric Analysis, Adaptivity, Multipatch discretizations*

In this talk, we propose a new approach for adaptive discretizations in Isogeometric Analysis. In order to avoid the non local refinement of tensor product discretizations in 2D or higher dimensions, we decompose the computational domain into multiple geometrically conforming patches. On each of these patches, we set up individual tensor product discretizations. Since we use different grid sizes on each patch we usually have non conforming but nested discretizations on the interfaces. The nesting property allows the coupling of local basis functions in a H^1 conforming way across interfaces. This also applies to T-junctions emerging from the local refinements.

We further give some insight on the computation and formation of a spline basis using a classical and a more algebraic approach and the problems that come with each of these approaches. Finally, we show some results by employing this method to a simple adaptive test problem, utilizing patchwise refinement and a residual a posteriori error estimator.

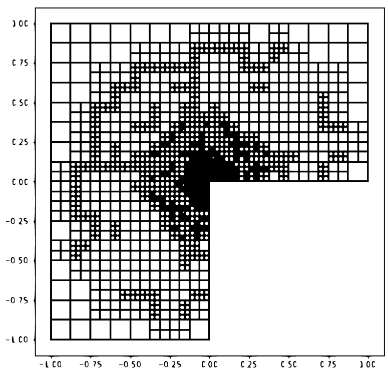

Figure 1: adaptive mesh refinement of an L-shape domain.

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DynAMO: Dynamic Anticipatory Mesh Optimization with Reinforcement Learning

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Key Words: *Adaptive Mesh Refinement, Hyperbolic Conservation Laws, Reinforcement Learning*

Adaptive mesh refinement (AMR) has been widely used in many problems, providing improved solutions compared to uniform mesh refinement with a similar number of degrees of freedom. However, applying AMR to time-dependent problems for each time step is computationally infeasible for simulations of meaningful size. One possible way to reduce the computational cost for the re-meshing step is performing re-meshing less frequently. However, as our adaptive mesh is constructed based on the current solution, it may not reflect the solution features in subsequent time steps. To overcome this challenge, we propose DynAMO, a dynamic anticipatory mesh optimization approach that uses multi-agent reinforcement learning. In DynAMO, each agent is associated with a mesh element and can take actions such as refining, de-refining, or doing nothing. By receiving rewards based on the next-regrid-time-step error distribution during training, the agents learn the dynamics of the problem and error propagation behavior over a longer period of time. Unlike traditional adaptive mesh refinement, this allows us to perform mesh optimization that reflects the solution features in future time steps. We demonstrate the effectiveness of DynAMO on hyperbolic conservation laws such as the advection equation and compressible Euler equations, showing that it can effectively capture solution characteristics and anticipate error propagation dynamics.

Best approximations of matrices and differential operators

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Key Words: *Best approximations, n-widths, isogeometric analysis*

It is well known that if the singular values of a matrix are distinct, then its best rank-n approximation in the Frobenius norm is uniquely determined and given by the truncated singular value decomposition. On the other hand, this uniqueness is in general not true for best rank-n approximations in the spectral norm. In this talk we relate the problem of finding best rank-n approximations in the spectral norm to Kolmogorov n-widths and corresponding optimal spaces [1]. By providing new criteria for optimality of subspaces with respect to the n-width, we describe a large family of best rank-n approximations to a given matrix. This results in a variety of solutions to the best low-rank approximation problem and provides alternatives to the truncated singular value decomposition. This variety can be exploited to obtain best low-rank approximations with problem-oriented properties.

We further discuss the generalization of these results to compact operators in L^2, and explain how they can be used to both describe the out-performance of smooth spline approximations of solutions to differential equations when compared to classical finite element methods [3], and to solve the outlier-problem in isogeometric analysis [2].

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The unique continuation problem for the heat equation discretized with a high-order space-time nonconforming method

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Key Words: *unique continuation, data assimilation, heat equation, discontinuous Galerkin*

We discretize a unique continuation problem subject to the heat equation and present the associated numerical analysis. This unique continuation problem consists in reconstructing the solution of the heat equation in a target space-time subdomain given its (noised) value in a subset of the computational domain. Both initial and boundary data can be unknown.

This problem is ill-posed and does not have a standard stability estimate. Instead, we have a *conditional* stability estimate, so that an a priori estimate on the solution is needed to bound its norm in the target subdomain, see for instance [2].

The considered discretization method uses discontinuous high-order polynomials in space and in time. The discrete equations are obtained by minimizing a Lagrangian functional. Moreover, some stabilization terms are considered to regularize the ill-posed problem.

We get optimal a priori error bounds for a weak (residual) norm. The conditional stability estimate of the continuous problem is then used to obtain a priori error bounds in energy norm. These error bounds optimally account for the ill-posedness of the continuous problem. The strategy used for the analysis has been introduced in [1] for the Laplace problem.

Some numerical experiments are presented to check the actual efficiency of the method.

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Poster Presentations

Adaptive radial basis function finite difference scheme for linear elasticity problems

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Key Words: *Radial basis function, Finite difference, Polyharmonic spline, Polynomial basis, Residual-based adaptivity, Linear elasticity*

In this research, the radial basis function finite difference method (RBF-FD) is further developed to solve one- and two-dimensional linear elasticity problems. The related differentiation weights are computed by applying the supplemented version of the RBF with a polynomial basis. The polyharmonic splines (PHS) are chosen for the type of the RBF, i.e., the combination of the odd m-order PHS $\phi(r) = r^m$ with additional polynomial functions up to degree p serves as the basis [1].

Within this concept, a new residual-based adaptive point-cloud refinement algorithm is presented and its computational performance is analyzed. The numerical efficiency of the PHS RBF-FD scheme is tested by the use of the relative errors measured in ℓ_2-norm on some representative benchmark problems. The examples range from problems with smooth and non-smooth solutions, applying the newly developed h-adaptive point-cloud refinement strategy [2].

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Mixed *hp* finite element method for singularly perturbed fourth order boundary value problems with two small parameters

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Key Words: *boundary layers, exponential convergence, fourth order singularly perturbed problem, mixed hp finite element method, uniform convergence*

We consider fourth order singularly perturbed boundary value problems with two small parameters, and the approximation of their solution by the *hp* version of the finite element method on the *spectral boundary layer* mesh from Melenk et al. We use a mixed formulation requiring only C^0 basis functions in two-dimensional smooth domains. Under the assumption of analytic data, we show that the method converges uniformly, with respect to both singular perturbation parameters, at an exponential rate when the error is measured in the energy norm. Our theoretical findings are illustrated through numerical examples, including results using a stronger (balanced) norm.

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The Singular Function Boundary Integral Method for solving three-dimensional Laplacian problems with conical vertex singularities

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Key Words: *Laplace Equation, Vertex Stress Intensity Factors, Singular Function Boundary Integral Method.*

The Singular Function Boundary Integral Method (SFBIM) [2, 3] is extended to solve three-dimensional Laplacian problems with conical vertex singularities. The solution is approximated by the leading terms of the local asymptotic series in spherical coordinates [4]. In order to calculate the unknown singular coefficients, i.e., the vertex stress intensity factors, the Laplacian problem is discretized by applying Galerkin's principle. The governing equation is weighted by the local functions over the domain and the volume integrals are then reduced to surface ones by means of Green's second identity. Given that the local solution satisfies identically the boundary conditions over the conical surface causing the vertex singularity, the dimension of the problem is reduced by one and the boundary integrals need to be calculated only far from the vertex singularity, which yields a considerable reduction of the computational cost. Neumann boundary conditions are weakly imposed and Dirichlet conditions are applied by means of Lagrange multiplier functions [2, 3]. The latter are approximated by means of finite elements over the corresponding boundary parts and the corresponding nodal values are thus additional unknowns. The method is applied to a test problem considered by Zaltzman and Yosibash [4]. As in previous studies with edge singularities [1], the method exhibits fast convergence and yields accurate estimates of the vertex stress intensity factors. The application of the method to other problems as well as its limitations are also discussed.

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hp Discontinuous Galerkin Finite Element Methods for the approximation of singularly perturbed boundary value problems with two small parameters

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Key Words: *singularly perturbed problem; Discontinuous Galerkin method; boundary layers;* hp *finite element method; exponential convergence*

We consider second order singularly perturbed boundary value problems with two small parameters, in one and two dimensions and we aim to describe an *hp* version of the finite element method using the Discontinuous Galerkin method. More specifically, we will solve those problems using the so-called *Spectral Boundary Layer* mesh, to achieve exponential convergence. Finally, our theoretical findings are illustrated through numerical examples.

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